Ten Conferences WORDS and their Contribution to the Field of Combinatorics on WORDS

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Abstract

This prologue summarizes the history of the conference WORDS, and the contributions which were presented, our goal being to testify how the conference may be embedded in the development of the field of Combinatorics on Words.

The representation of numbers by sequences of symbols is inherent in mathematics. A noticeable step has certainly been reached by the introduction of the decimal representation. This notion appears at the tenth century in the documents written by the Arabian mathematician Al-Uqlidisi, who was interested in the Indian system of numeration. In the occident, the fractional representation of numbers delayed the introduction of the decimal representation until the seventeenth century, when the Belgian mathematician Simon Stevin recommended it as a performing tool of calculation. In his document “La Disme", he predicted that these methods of calculation would be extended to unrestricted representations and even be applied to the so-called incommensurable numbers.

A systematic study of words as formal mathematical objects appeared at the beginning of the twentieth century, when three now famous papers were published by the Norwegian mathematician Axel Thue [18, 19, 20]. Presently, thanks to Jean Berstel [1] and James F. Power [17], we obtain translations of these papers in more recent terminology and in relation to more contemporary directions of research.

Actually, from the end of the nineteenth century, words have taken an important role in different domains of mathematics such as Groups, Semigroups, Formal Languages, Number Theory and Ergodic Theory. On the other hand, constituting a unified treatment of words was more and more in demand: such a request was especially stimulated by Marcel-Paul Schützenberger, who presented a series of challenging questions concerning the topic in his lectures of the fifties. It is also to be mentioned that around the same time a Russian school of the theory, maybe a bit more implicitly, was initiated in Moscow by Petr S. Novikov and Sergei I. Adian.

In the growing importance of the field of Combinatorics on Words, a new fundamental step was reached in the eighties which was published in the series of Lothaire's books [9, 10] and the famous “Theory of Codes" from Jean Berstel and Dominique Perrin [2] (cf also [3]). The importance of the topic has also been supported by the publication of the third book from Lothaire [11], which testifies that words have fundamental applications in many domains of computer science. Another important step was reached in 1991, when the terminology “Combinatorics on Words" was introduced in the famous “Mathematics Subject Classification" as a subfield of “Discrete Mathematics in Relation to Computer Science" (68R15).

At the occasion of the publication of the first Lothaire's book, Dominique Perrin organized the meeting “La Fête de Mots" in Rouen in 1985, where new fascinating results and challenging questions were presented. We all know that in view of sharing scientific results, international workshops play a complementary part beside the publication of books and full papers: they quickly provide a picture of the state-of-the-art and are special meeting places for the community. Actually, until the end of the nineties, due to the numerous varied topics in theoretical computer science, in most of the international conferences only a few sessions could be granted to Combinatorics on Words: with regard to the growing number of papers concerned by the topic, a specific series of international conference essentially devoted to words was strongly required. The organization of “WORDS" was the response to such a request: with Aldo de Luca and Antonio Restivo, we drew the main features of the project, and opted in particular for a bi-annual series of meetings. The first conference WORDS was planned in Rouen, France, during September 1997. Thanks are due to the many researchers that supported that event and the subsequent conferences (they will recognize themselves); a special thought is due to Jean Berstel, for his invaluable investment in the project. A series of ten international workshops was organized:

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WORDS 2001, Palermo, Italia, Filippo Mignosi chair,
WORDS 2003, Turku, Finland, Juhani Karhumäki chair,
WORDS 2005, Montréal, Canada, Srečko Brlek and Christophe Reutenauer co-chairs,
WORDS 2007, Marseille, France, Srečko Brlek and Julien Cassaigne co-chairs,
WORDS 2009, Salerno, Italia, Arturo Carpi and Clelia De Felice co-chairs,
WORDS 2011, Prague, Czech Republic, Štěpán Holub and Edita Pelandova co-chairs,

WORDS 2013, Turku, Finland, Juhani Karhumäki and Luca Zamboni co-chairs,
WORDS 2015, Kiel, Germany, Florin Manea and Dirk Nowotka, co-chairs.

For each meeting, papers selected to be presented were published in proceedings; moreover among them, some of the most stricking were published in a special issue of an international scientific journal [4, 5, 6, 8, 13, 14, 15, 16].

The tenth conference WORDS provides the opportunity to take a panoramic look at all the fascinating papers which were presented. Of course, drawing up an exhaustive list would be an impossible task: in what follows, only a limited number of results will be mentioned: according to the frequency of their presentations, we have opted for a classification into three thematic areas.

The topic of patterns

The famous infinite word of Thue-Morse has the fundamental property that given a letter a and an arbitrary word v no factor of type $avava$ may appear in the infinite word. It is also well-known that this word allows one to construct an infinite cube-free word on a three letter alphabet. These properties have naturally opened a more general problem: given a finite alphabet does an infinite word exist such that none of its factors may be a *repetition* of type $(uv)^k u$, with $u \neq \varepsilon$ and $k \geq 2$?

- *Avoidance of patterns* is a central question in the topic. In the meeting of 1997, Roman Kolpakov, Gregory Kucherov and Yuri Tarannikov presented some properties of repetition-free binary words of minimal density. During that of 2003, James Currie composed an extensive survey concentrating on open problems. In the same workshop, Ina N. Rampersad, Jeffrey Shallit and Ming-Wei Wang were interested in infinite words avoiding long squares. In the conference of 2007, Arturo Carpi and Valerio D'Alonzo exhibited a word avoiding near repeats. In their contribution at WORDS 2011, Elena A. Petrova and Arseny M. Shur focussed on binary cube-free words that cannot be infinitely extended preserving cube-freeness: they prove that such words exist but can have arbitrarily long finite cube-free extensions both to one side and two sides. In WORDS 2013, Tero Harju presented an infinite square-free word on a three-letter alphabet that can be shuffled with itself to produce another infinite square-free word. In the same conference, Tomi Kärki gave an overview of the results concerning repetition-freeness in connection with the so-called *similarity relations*, which are relations on words of equal length induced by a symmetric and reflexive relation on letters. The set of square-free words over a given alphabet may be represented by a prefix tree whose nodes are these words themselves: in their presentation at WORDS 2015, Elena Petrova and Arseny Shur contributed to studying the structure of that tree in the case of ternary square-free words.

- A natural question consists in examining the number of patterns that may appear in a finite word. From this point of view, Aviezri Fraenkel and Jamie Simpson showed that the number of different squares in a word is bounded by $2n$, and stated the so-called “square-conjecture" that stipulates that the bound is in fact n. In their contribution at WORDS 1997, the exact number of squares in the Fibonacci word f_n was shown to be equal to $2(|f_{n-2}| - 1)$. With regard to arbitrary words, the bound was refined to $2n - O(\log n)$ by Lucian Illie in his talk at WORDS 2005. It is also conjectured that binary words have the largest square density: in their contribution at WORDS 2015, Florin Manea and Shinnosuke Seki solved this new conjecture: more precisely, they showed the irrelevance of the alphabet size in proving the preceding square-conjecture as long as the alphabet is not unitary.

- A *run* may be defined as the occurrence of a repetition of exponent at least 2 that is maximal in the sense that it cannot be extended to the left or right to obtain the same type of pattern. Such objects play an important role in a lot of string matching algorithms. In 1999, Roman Kolpakov and Gregory Kucherov conjectured that, for a word of length n the number of runs is bounded by n. In their talk at WORDS 2009, Maxime Crochemore, Lucian Ilie and Liviu Tinta proved that the number of its runs is bounded above $1.029n$.

- The so-called *suffix-square completion* allows the derivation of a word w, in any word wx, such that w has a suffix of type yxy. In their talk at WORDS 2015, Marius Dumitran and Florin Manea make use of such an operation for generating infinite words that do not contain any repetition of exponent greater than 2.

- The *repetition threshold* for k letters, which we denote by $RT(k)$, is the smallest rational number α, such that there exists an infinite word whose finite factors have exponent at most α. Actually, repetitions of the Thue-Morse sequence have exponent at most 2 and $RT(2) = 2$. In the seventies, Françoise Dejean conjectured that for every $k > 2$ the following holds:

$$RT(k) = \begin{cases} 7/4 \text{ if } k = 3 \\ 7/5 \text{ if } k = 4 \\ k/k - 1 \text{ otherwise.} \end{cases}$$

A word is h^+-free if it does not contain a repetition of exponent h' with $h' > h$. A stronger version of Dejean’s conjecture was stated by Pascal Ochem in his contribution at WORDS 2005:

- For every $k \geq 5$, an infinite $(k/k - 1)^+$-free word over k letters exists with letter frequency $1/k + 1$.

- For every $k \geq 6$, an infinite $(k/k - 1)^+$-free word over k-letter exists with letter frequency $1/k - 1$.

Dejean's conjecture had been partially solved by several authors. The final proof was completed in 2009 by James Currie and Narad Rampersad for $15 \leq n \leq 26$, and independently by Michaël Rao for $8 \leq k \leq 38$. In WORDS 2009, Michaël Rao presented his proof: in fact the technique that he applied allowed him to prove Ochem's stronger version of the conjecture for $9 \leq k \leq 38$; moreover in a private communication at WORDS 2015, he announced the complete resolution of this last conjecture.

Starting with $RT(k)$, the definition of $FRT(k)$, the *finite repetition threshold* for k letters, stipulates that only a finite number of factors with exponent α may exist in the corresponding infinite word. In 2008, Jeffrey Shallit showed that $FR(2) = 7/3$. In their presentation at WORDS 2011, Golnaz Badkobeh and Maxime Crochemore proved that $FRT(3) = RT(3) = 7/4$, and they conjectured that $FR(4) = FRT(4) = 7/5$. This conjecture was solved by Golnaz Badkobeh, Maxime Crochemore and Michaël Rao, which moreover established that $FR(k) = FRT(k)$ for $k \leq 6$ (private communication during WORDS 2015).

- Abelian patterns are also concerned: an *abelian square* consists of a pattern of type xy, where the word y is obtained by applying a permutation on the letters of x. In 1992, Veikko Keränen solved a famous open problem by constructing an *abelian square free* word over a four-letter alphabet. In the meeting of 2007, he presented new abelian square-free morphisms and a powerful substitution over 4 letters.

A word of length n can contain $O(n^2)$ distinct abelian squares: in their talk of WORDS 2015, Gabriele Fici and Filippo Mignosi studied infinite words such that the number of abelian square factors of length n grows quadratically with n.

Two words u, v are *k-abelian equivalents* if every word of length at most k occurs as a factor in u as many times as in v and the prefixes (suffixes) of length $k-1$ of u and v are identical. Clearly the classical notion of abelian equivalence corresponds to $k = 1$. A word is *strongly k-abelian nth-power* if it is k-abelian equivalent to an nth-power. In WORDS 2013, Mari Huova and Aleksi Saarela proved that strongly k-abelian nth-powers are unavoidable on any alphabet. In his talk of WORDS 2015, Michäel Rao presented techniques to decide whether a morphic word avoids abelian (k-abelian) repetitions. In particular this led him to prove that long abelian squares are avoidable on a ternary alphabet: a positive answer to a weak version of a question from Mäkelä.

- Pattern avoidance by palindromes was the subject of the talk from Inna A. Mikhailova and Mikhail Volkov, in WORDS 2007.

A word is a *pseudopalindrome* if it is the fixed point of some involutary antimorphism ϕ of the free monoid (i.e. $\phi^2 = id$, $\phi(uv) = \phi(v)\phi(u)$) and the *pseudopalindromic closure* of a word w is the shortest pseudopalindrome having w as a prefix. In their contribution at WORDS 2009, Damien Jamet, Genevieve Paquin, Gwenaël Richomme and Laurent Vuillon present several combinatorial properties of the fixed points under iteration by pseudopalindromic closure.

Complexity issues

In the literature, several notions of complexity can be associated with a word, the most famous being the *factor complexity*: given a word w, this complexity measures the number $p_w(n)$ of different factors of length n occurring in w. The famous characterization of Morse-Hedlund for ultimately periodic words has led to the introduction of the infinite *Sturmian words* whose complexity is $p_w(n) = n + 1$, the best known example of them being the famous *Fibonacci word*.

- Other notions of complexity for infinite words were defined by Sébastien Ferenczi and Zoltán Kása in their contribution at WORDS 1997: the behavior of *upper (lower) total finite-word complexity* and *upper (lower) maximal finite-word complexity* were compared to the classical factor complexity, moreover new characterizations of Sturmian sequences were obtained.

- The *recurrence function* was introduced by Morse and Hedlund: given a factor u, it associates with every non-negative integer n the size $R_u(n)$ of the smallest window that contains every factor of length n of u. During his talk at WORDS 1997, Julien Cassaigne introduced the *recurrence quotient* as $\limsup\limits_{n\to\infty} \frac{R(n)}{n}$, moreover he computes it for Sturmian sequences.

- The notion of *special factor* allows one to obtain a performing characterization of Sturmian words. In the case of finite words, it leads to the introduction of two parameters, namely R, L which, given a finite word w, represent the least integer such that no right (left) special factors of length $\geq R$ ($\geq L$) may occur in w. In his contribution at WORDS 1997, Aldo de Luca studied the connections between these parameters and the classical factor complexity.

- The study of the ratio $p(n)/n$ brings significant information on infinite words. In WORDS 1999, Alex Heinis showed that if $p(n)/n$ has a limit, then it is either equal to 1, or more than to 2. By using the Rauzy graphs, in WORDS 2001, Ali Aberkane presented characterizations of the words such that the limit is 1.

- A word is *balanced* if for any pairs (u, v) of factors with same length, and for any letter a, we have $||u|_a - |v|_a| \leq 1$ (where $|u|_a$ stands for the number of occurrences of the letter a in u). In the paper he presented in WORDS 2001, Boris Adamczewski defined the *balance function* as $\max\limits_{a\in A} \max\limits_{u,v\in F(w)} \{||u|_a - |v|_a|\}$: with regard to the so-called *primitive substitutions*, he investigated the connections between the asymptotic behavior of the balance function and the incidence matrix of such a substitution. In the workshop of 2007, Nicolas Bédaride, Eric Domenjoud, Damien Jamet and Jean-Luc Remy studied the number of balanced words of given length and height on a binary alphabet. During WORDS 2013, two studies relating investigations of the property of balancedness of the Arnoux-Rauzy words were presented: one was due to Julien Cassaigne and the other one to Vincent Delecroix, Tomáš Hejda and Wolfgang Steiner.

- The *arithmetical complexity* of an infinite sequence is the number of all words of a given length whose symbols occur in the sequence at positions which constitute an arithmetical progression. Let us consider Toeplitz words as infinite words generated by iteratively substituting the symbol "?" for the word w in an infinite periodic word $w^{\omega} \in (A \cup \{?\})^{\omega}$. In her talk at WORD 2003, Anna Frid showed that non-periodic uniformly recurrent words whose arithmetical complexity grows linearly are precisely Toeplitz words of a specific form. The study of arithmetical complexity also appeared in the contribution of Julien Cassaigne and Anna Frid at WORDS 2005: they gave a uniform $O(n^3)$ upper bound for the arithmetical complexity of a Sturmian word and provided explicit expressions for the arithmetical complexity of Sturmian words of slope between $1/3$ and $2/3$ (this condition is in particular satisfied by the infinite Fibonacci word). In such a case the difference between the genuine arithmetical complexity function and the precedingly mentioned upper bound is itself bounded and ultimately 2-periodic.

Given an arbitrary finite word $w = w_0 w_1 \cdots$ (each factor w_i being a letter) and three integers k, d, c, the so-called *arithmetic progression* of length k with starting number c and difference d is the word $w_c w_{c+d} \cdots w_{c+(k-1)d}$. This progression is *homogeneous* if a letter α exists such that $w_{c+id} = \alpha$ $(0 \leq i \leq k-1)$. In her presentation at WORDS 2015, Olga G. Parshina considered the so-called *generalized Thue-Morse word*: given the alphabet $A = \{0, \cdots q-1\}$, the letter w_i is defined as the sum modulo q of the digits of i in base q, i.e. $w_i = (\sum_{0 \leq j \leq n-1} x_j) \bmod q$, with $i = \sum_{0 \leq j \leq n-1} x_j q^j$. In the case of a 3-letter alphabet, she proved that the maximal length of a homogeneous arithmetic progression of a given difference, i.e. $\mathcal{A}(d) = \max\{k | c \geq 0\}$, is expressed by the following formula:

$$\max_{d<3^n} \mathcal{A}(d) = \begin{cases} 3^n + 6, & \text{if } n = 0 \bmod 3 \\ 3^n, & \text{otherwise,} \end{cases}$$

this maximal value corresponding to $d = 3^n - 1$.

- The *palindromic complexity* of an infinite word is the function which counts the number $P(n)$ of different palindromes of each length occurring as factors in the word. In their talk at WORDS 2005, Peter Baláži, Zuzana Masáková and Edita Pelantová provided an estimate of the palindromic complexity $P(n)$ for uniformly recurrent words; denoting by $p(n)$ the classical factor complexity, this estimation is based on the equation: $P(n) + P(n+1) = p(n+1) - p(n) + 2$.

- The *m-binomial complexity* of an infinite word w maps an integer n to the number of m-binomial equivalence classes of factors of length n that occur in w. This relation of binomial equivalence is defined as follows: two words u, v are

m-equivalent if, for any word x of length at most m, x appears in u and v with the same number of occurrences. In their contribution at WORDS 2013 Michel Rigo and Pavel Salimov computed the m-binomial complexity of famous words: Sturmian words and the Thue-Morse word.

- Given a word v with several occurrences in an infinite word, the set of *return words* of v has for elements all distinct words beginning with an occurrence of v and ending just before the next occurrence of v. It had been proved that a word is Sturmian if and only if each of its factors has two returns: in the meeting of 2011, Svetlana Puzynina and Luca Zamboni proved that a word is Sturmian if and only if each of its factors has two or three abelian returns.

Factorization of words. Equations

Further important information may be obtained by decomposing a word into a convenient sequence of consecutive factors: $w = w_1 \cdots w_n$.

- In their talk at WORDS 1997, Juhani Karhumäki, Wojciech Plandowski and Wojciech Rytter investigated the properties of the so-called $\mathcal{F}$-factorization, where the preceding sequence $(w_1, \cdots, w_n)$ satisfies a given property $\mathcal{F}$. From an algorithmic point of view, they examined the behavior of three fundamental properties of this factorization, namely *completeness*, *uniqueness* and *synchronization*.

- By making use of the so-called *closed words*, Alessandro De Luca and Gabriele Fici prove in WORDS 2013 that after swipping its first letter every standard Sturmian words can be written as an infinite product of squares of reversed standard words.

- *Periodicity* is also clearly concerned with the notion of factorization. With the preceding notation, if for an integer $n \geq 2$, all the words $w_1, \cdots, w_{n-1}$ are equal, the word w_n being one of their prefixes, we say that the length of w_1 is a *period* of w. The famous theorem of Fine and Wilf states that if some powers of two words x, y have a common prefix of length $|x| + |y| - \gcd(|x|, |y|)$, then x and y themselves are powers of the same word. In WORDS 1997, Maria Gabriella Castelli, Filippo Mignosi and Antonio Restivo presented an extension of Fine and Wilf's theorem for three periods. In their contribution at WORDS 2003, Sorin Constantinescu and Lucian Ilie proved a new extension of that theorem for arbitrary number of periods, and in WORDS 2007, Vesa Halava, Tero Harju, Tomi Kärki and Luca Q. Zamboni studied the so-called relational Fine and Wilf words. In his contribution at WORDS 2007, Kalle Saari examined periods of the factors of the Fibonacci word.

- A word w is *quasiperiodic* if another word x exists such that any position in w falls within an occurrence of x as a factor of w (informally, w may be completely "covered" by a set of occurrences of the factor x). In WORDS 2017, Florence

Levé and Gwenaël Richomme presented some properties of quasiperiodic episturmian words. In the conference of 2013, they provided algorithms for deciding whether a morphism is strongly quasiperiodic on finite and infinite words.

- In WORDS 1999 Juhani Karhumäki and Ján Maňuch proved that if a nonperiodic bi-infinite word contains three disjoint factorizations on the words of a prefix-free set X, then a set Y with cardinality at most $|X| - 2$ exists such that $X \subseteq Y^*$. Such a type of *defect effect* is strongly connected to *independent systems of equations*, as illustrated by the paper presented by Tero Harju and Dirk Nowotka at WORDS 2001, where the case of equations in three variables was investigated. Some properties of infinite systems of equations were presented by Štěpán Holub and Juha Kortelainen at WORDS 2005.

- The famous Post Correspondence Problem (*PCP* for short) is also connected to decomposition of words. Given two morphisms h, g, it considers the equation $h(x) = g(x)$ in asking whether the existence of a solution distinct of the empty word is decidable. In the meeting of 2005, Vesa Halava, Tero Harju, Juhani Karhumäki and Michel Latteux provided an extension of the decidability of the marked *PCP* to instances with unique blocks. The properties of new variants such as the circular-*PCP*, and the n-permutation *PCP* were also examined by Vesa Halava in his presentation at WORDS 2013. In the same meeting, in the topic of the *Dual-PCP*, the so-called periodicity forcing words was the aim of the talk from Joel D. Day, Daniel Reidenbach and Johannes C. Schneider.

- It is noticeable that, with respect to the prefix ordering, six minimal squares exist such that every infinite Sturmian word s may be written as a product of these squares, say $s = X_1^2 X_2^2 X_3^2 \cdots$ With such a condition, the *square root* of s may be defined as $\sqrt{s} = X_1 X_2 X_3 \cdots$ and in their presentation at WORDS 2015, Jarkko Peltomäki and Markus Whiteland proved that $\sqrt{s}$ itself remains a Sturmian word with same slope as s.

- Given an alphabet A, a subset X of A^n is *representable* if it occurs as the set of all factors of length n of a finite word. Clearly, the set A^n itself is represented by any De Bruinj word of order n. One of the questions presented in the talk given by Shuo Tan and Jeffrey Shallit at WORDS 2013 was to examine the number of different subsets of A^n which are representable: they proved that in the binary case this number belongs to the interval $[2^{2^n}, (\sqrt[10]{4})^{2^n}]$.

Many other interesting topics were presented during these ten conferences: in this survey we can only mention some of them.

With regard to sets of words, in WORDS 1999, Jean Berstel and Luc Boasson proved that, given a finite set of words S, at most one (normalized) multiset P may exist such that S is the shuffle of the words in P, the multiset P being effectively computable. Codes were also the subject of talks from Véronique Bruyère and Dominique Perrin (WORDS 1997), Jean Néraud and Carla Selmi (WORDS

2001), Fabio Burderi (WORDS 2011) and Dominique Perrin (WORDS 2015).

Extensions of the classical concept of words were the subject of a lot of presentations. The notion of a partial word was introduced in WORDS 1997 by Jean Berstel and Luc Boasson; in WORDS 2009, in the framework of binary words, Francine Blanchet Sadri and Brian Shirey examined the relationship between such a notion and periodicity. Investigating the properties of multidimensional words was the aim of the talk from Valérie Berthé and Robert Tijdeman at WORDS 1999, and in the meeting of 2005, it was also the subject of two presentations: one was given by Pierre Arnoux, Valérie Berthé, Thomas Fernique and Damien Jamet, and the other one by Jean-Pierre Borel.

Connections with Semigroups Theory were the subject of the lectures of Sergei I. Adian at WORDS 2003.

Words in connection with Number Theory and Numeration Systems were also the features of very interesting contributions from Tom Brown (WORDS 1999), Petr Ambrož and Christiane Frougny (WORDS 2005), Daniel Dombek (WORDS 2011), Shigeki Akiyama, Victor Marsault and Jacques Sakarovitch (WORDS 2013).

Evidently each of the numerous results which were presented in the ten conferences plays a noticeable part in the state-of-the-field. We will have achieved our goal if theses notes testify to the role of the WORDS conferences in the development of the field of Combinatorics on Words.

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